Analytic solution to find optimal balance between customer acquisition and retention spending.

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Solution analytique pour trouver le meilleur équilibre entre les dépenses d'acquisition et rétention de clientèle

Résumé

Trouver le meilleur équilibre entre les efforts d'acquisition et de rétention des clients est un problème important de CRM. Blattberg et Deighton (1996) (BD) ont donné une solution essentiellement graphique à ce problème pour un comportement dynamique de type "lost for good" du client et pour des taux de rétention constants. Leur solution a récemment été mise en cause par Pfeifer (2005) qui utilise une approche numérique. Ce papier trouve une fonction de réponse du client pour donner une solution analytique au problème BD. Cela facilite la comparaison de ces méthodes d'optimisation alternatives et permet de donner une réponse systématique à la question s'il est optimum de dépenser plus en rétention pour des clients plus économiques à retenir que d'acquérir.

Mots clés: CRM, Lifetime Value, rétention des clients, acquisition des clients, optimisation

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Abstract

Balancing customer acquisition and retention efforts is an important CRM problem. Blattberg and Deighton (1996) (BD) gave a mainly graphic solution to this problem for the constant retention rate "lost for good" customer dynamic behaviour. That solution has been recently challenged and modified by Pfeifer (2005) using a numeric approach. This paper finds a proxy customer response function in order to give an analytic solution to the BD problem. This facilitates comparison of those alternative optimisation methods and gives a systematic answer to the question whether for customers cheaper to retain than to acquire it is optimal to spend more on retention.

Key words: CRM, Lifetime Value, Customer Retention, Customer Acquisition, Optimisation

INTRODUCTION

Balancing customer acquisition and retention efforts is an important CRM problem. Blattberg and Deighton (1996) using a simple customer dynamic behaviour model, determine, the optimal amount of acquisition spending and retention spending independently. Berger and Nasr-Bechwati (2001) extend this approach to a constrained allocation of promotional spending between acquisition and retention. The latter models are applied by Berger and Bernstein (2002) to real-world applications in the diagnostic self-testing industry. All these optimisation procedures use customer lifetime value (CLV) and the derived concept of customer equity (CE) (Blattberg and Deighton 1996; Blattberg, Getz and Thomas 2001; Rust, Zeithaml and Lemon, 2000) as the basis. CLV has been fundamental for many years in direct and database marketing as mentioned by Dwyer (1989) and discussed by Keane and Wang (1995) in the publishing industry.

Berger and Nasr (1998) examined a series of models to compute CLV for different situations in terms of revenue streams, costs and probabilities. Following papers added modified CLV formulas (Mülhern,1999; Reinartz and Kumar, 2000; Rust, Zeithaml and Lemon, 2000)

Jain and Singh (2002) in their frequently cited review of the CLV literature identify three main research directions. The first is the development of CL*V calculation models* that focus on the revenue stream from customers and on acquisition, retention and other marketing costs in order to facilitate calculations, resource allocation and optimisation of CLV. The second research stream described as *customer base analysis concentrates* on methods that analytically predict the probabilistic value of customer transactions from the existing customer base. The last direction uses analytical models in a normative way it analyses *CLV implications for managerial decisions*.

In this paper we focus on finding optimal balance between customer acquisition and retention spending for the BD problem by maximising CLV and CE. We find an analytical solution to optimize retention spending for this problem as an alternative to the original graphical solution and to its corollary numeric solution. As the original optimisation procedure suggested by Blattberg and Deighton (1996) has been recently challenged by Pfeifer (2005) we use the computational rapidity of our analytic solutions in order to systematically compare the two alternative optimisation procedures, to perform sensitivity analysis and to derive and discuss rules that help guide managerial decisions.

THE BLATTBERG AND DEIGHTON MODEL

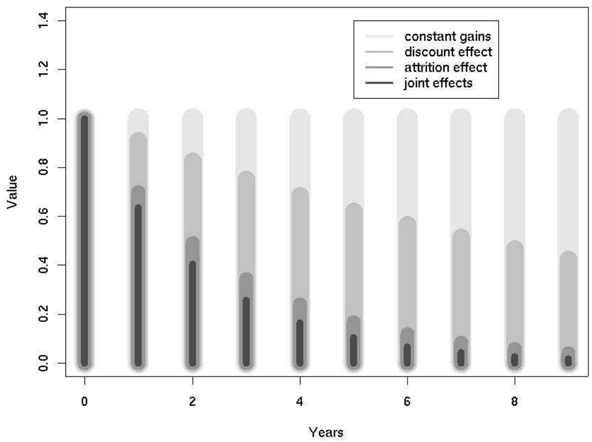
Blattberg and Deighton proposed a model for helping managers determine the optimal balance between acquisition and retention spending. They based their reasoning on CLV calculations using a simple model of dynamic customer behaviour known in the more recent literature as the retention model. This dynamic behaviour, found according to Dwyer (1989) in many industries like banking and insurance services, press subscriptions etc., supposes that repeat buying defines an active customer. In other words when a customer doesn't renew his purchase or contract, he is "lost for good" and when he comes back after one or several periods of absence he is treated as a new customer[i]. The *retention model* relies upon the conventional assumption that after customers are lost, they do not return (Rust, Lemon, and Zeithaml 2004). The evaluation of customer potential in a relation of this kind will only take into account the customer's *probability to remain active* from one period to another or what Blattberg and al. (2001) call the customer's *survival probability*. The customer lifetime corresponds to the number of successive periods during which the customer is and remains active.

It can be shown that at constant retention rate and constant gain per purchase an initial 1€ gain from a customer is worth on the long term (1+d)/(1+d-r) €, where r and d are retention and discount rates. This means for example that the lifetime value (or rather long term value) of a customer initially 1€ worth is 3€ if the retention rate is 80% and discount rate is 20%. This formula captures two essential phenomena to be taken into account when valuing a customer. One is the attrition that occurs during the whole lifetime of a cohort and produces a survival rate after t periods worth r^t. The other one is the need to discount future revenues. A 1 € gain obtained after t years is worth no more than $1/(1+d)^t$, where d is the discount rate, a measure that is near but with an inverse impact to the interest rate. Table 1 and Figure 1 illustrate this logic guiding lifetime value calculation.

Table 1 - Attrition and discount effects on future gains

t	gain	discount	survival	joint
	g	$1/(1+d)^t$	r^t	$(r/(1+d))^t$
0	1.00	1.00	1.00	1.00
1	1.00	0.83	0.80	0.67
2	1.00	0.69	0.64	0.44
3	1.00	0.58	0.51	0.30
4	1.00	0.48	0.41	0.20
5	1.00	0.40	0.33	0.13
6	1.00	0.33	0.26	0.09
7	1.00	0.28	0.21	0.06
8	1.00	0.23	0.17	0.04
9	1.00	0.19	0.13	0.03

Figure 1 - Attrition and discount effects on future gains



Any future gain generated by a customer will be jointly affected by attrition and discount rate and needs to be corrected by a factor of $(r/(1+d))^t$ and if this gain *g* is constant during each period of the customer's lifetime, the discounted sum of these gains on the long term or the customer lifetime value (CLV) is

$$\sum_{t=0}^{\infty} g\left(\frac{r}{1+d}\right)^t = g\frac{1+d}{1+d-r}. \qquad (1)$$

If excluding the initial phase t=0 which can be seen as the acquisition phase then the value or the sum of gains of a newly acquired customer is

$$\sum_{t=1}^{\infty} g\left(\frac{r}{1+d}\right)^t = g\left(\frac{1+d}{1+d-r}-1\right) = g\frac{r}{1+d-r} \qquad (2)$$

The gains generated by a customer in a simplified retention model during each period are *(m-R/r)* (see figure 2) where *m* is the constant margin, *R* the retention cost per targeted customer and *r* the constant retention rate. *R* is the cost of the marketing effort that is necessary in order to maintain a constant retention rate of *r*. (m-R/r) measures customer profitability at each period. It indicates that margin (*m*) is collected from customers remaining after a retention campaign while retention costs apply to all targeted customers at the beginning of each campaign.

At t>0 retention costs (R) are incurred for all targeted customers present in a cohort (at time t) while margin (m) is collected only from those customers who will have responded, meaning those customers who will still be there at the end of the period or at the beginning of the next period (at time t+1)). *R/r* is therefore the *cost per retained customer*. As the amount and the value of such periodic gains diminishes due to customer attrition and the need to discount future revenues by a discount rate *d,* in order to compute the long term value they must be multiplied by the deflated sum of customer survival rates $\sum_{t=1}^{\infty}\left(\frac{r}{1+d}\right)^{t}$ = r/(1+d-r). The lifetime value of a just retained customer or what Blattberg et alii. (2001) tend to call Retention equity[ii] is then:

$$CLV = (m - R/r)\frac{r}{1+d-r} \qquad (3)$$

In order to get the complete image of the customer relationship profitability, customer acquisition phase when prospects are transformed into customers cannot be ignored. By analogy to retention gains, acquisition gain becomes *m-A/a* that is the margin (*m*) minus customer acquisition cost (A/a). A sequential, chronological representation of all gains of a customer's «lifetime» allows for better understanding of the long term values of a prospect, a new customer, a just acquired customer or an existing customer (figure 2.)

Figure 2. Timeline of gains generated by a customer/prospect

t	***0***	***1***	***2***	**...**	**∞**
gain by period	*-A/a+m*	-R/r+m	-R/r+m		-R/r+m

acquired prospect--> just acquired customer-->

new Cust. --> ex isting cust.-->

According to the stage the prospect/customer is in at the moment of calculation, several formulations of his Expected Lifetime Value (or long term value) are possible (see table 2).

Tableau 2 – Calculations of the Expected Lifetime Value according to the stage the prospect/customer is in (adapted from Pfeifer et alii, 2005)

Perspective	Expected Lifetime Value
Prospect	$a\left[-A/a+m+(m-R/r)\frac{r}{1+d-r}\right]$
New (t=0) or existing (t>0) customer	$m+(m-R/r)\frac{r}{1+d-r}$
Just acquired customer (t=0)	$(m-R/r)\frac{r}{1+d-r}$

They differ essentially in whether they include the initial margin or not. In this paper we use the CLV (customer lifetime value) notation (formula 3) for the just acquired customer lifetime value which doesn't include the initial margin. The other formulations in table 2, the new and existing customer lifetime values become then $(m+CLV)$. As these formulations don't include acquisition costs, Blattberg and Deighton (1996) introduce the term Customer equity (CE).

$$CE=(m+CLV)-A/a. \quad (4)$$

In order to find optimal acquisition spending this concept needed to be put in a prospect perspective to compute what could be called prospect lifetime value (PLV) or as Pfeifer (2005) suggests EPLV (expected prospect lifetime value).

$$PLV = a\,CE = a(-A/a+m+CLV) = a\left[-A/a+m+(m-R/r)\frac{r}{1+d-r}\right] \quad (5)$$

Thus the lifetime value of a prospect who is going to become a customer is the acquisition rate times the expected value of a newly acquired customer minus the acquisition spending. Formula (5) is essential to the Blattberg and Deighton (BD) model and a starting point in the BD approach to balance the trade-off between acquisition and retention spending. The BD approach has been recently challenged by Pfeifer (2005). In this paper we thoroughly compare the two approaches, analyse their differences and suggest further extensions.

OPTIMISATION PROCEDURES BALANCING ACQUISITION AND RETENTION COSTS COMPARED

The BD model expresses the retention probability as a function of the marketing effort (*R*) directed towards the customers being targeted. A modified exponential function of the retention budget is used whose form is consistent with an assumption of strictly diminishing returns to retention spending.

$$r = c_r\,[1 - exp(-k_r R)] \tag{6}$$

The ceiling c_r and the sensibility of a customer to retention effort k_r can be estimated by decision calculus using subjective response estimations from mangers (Berger & Nasr, 1998). Parameter c_r, the retention ceiling rate, is the manager's direct assessment of the maximum proportion of targeted customers that would be retained if there were no limit to spending. Parameter k_r can next be determined once the manager gives the current retention spending level and the current retention rate as $\log(c_r/(c_r - r))/R$.

The fact that the retention curve goes through the origin is seen as a weakness of the BD model by Pfeifer (2005) if the firm expects to retain some percentage of customers even in the absence of retention spending. We show that this "weakness" can be easily overcome without influence on the fundamental results of the model by introducing an additional parameter in the response function that does not affect marginal retention cost.

$$r = f_r + (c_r - f_r)\,[1 - exp(-k_r R)] \tag{7}$$

Parameter f_r the retention floor rate, is the managers direct assessment of the proportion of targeted customers that would be retained in absence of retention spending, and can be seen as a measure of customer inertia.

By introducing (7) or (6) in (3) the retention costs that maximise CLV can be found. As there is no closed form expression in order to find the optimal retention spending R*, Blattberg and Deighton suggest to find it graphically.

Similarly to the retention response function, the model assumes that acquisition rate *a* is a decelerating function of acquisition spending per prospect (A):

$$a = c_a\,[1 - exp(-k_a A)] \tag{8}$$

Where c_a is the ceiling and k_a is the sensibility of prospects to acquisition efforts.

To find the optimal balance between customer acquisition and retention spending Blattberg and Deighton (1996) suggest to start with maximising, what we call, the *prospect acquisition value* $a(m-A/a)$ using formula (8) to express the prospects acquisition response as a function of marketing efforts. They then use the resulting optimal acquisition rate a' to find the optimal *retention equity* in a prospect's perspective (see table 3). A generic formulation of this two step optimisation procedure is given in formula 9:

$$\underbrace{a(A)(m - A / a(A))}_{\max(1) \to a'} + \underbrace{a'(m - R / r)\frac{r}{1 + d - r}}_{\max(2)} \qquad (9)$$

This procedure is criticised by Pfeifer (2005) who considers it "unnecessarily myopic". In his view acquisition expenses are "suboptimal" when considering the smaller immediate gain (the margin *m* produced by an acquired customer) instead of taking into account the bigger long term value of a new customer *(m+ CLV)* (see table 3). It is the *long term value of a prospect to be acquired a(m+CLV) – A* and not that prospect's *acquisition value (am – A)* that should be maximised. Without changing the original formulation of the model (equation 5) he reverses the original two-step optimisation procedure by first finding the retention spending (R*) that maximises the new customer's lifetime value and then using it in the next step to find the optimal acquisition spending (A*), as in formula 10.

$$\underbrace{a(A)\underbrace{\left(m + (m - R / r)\frac{r}{1 + d - r} \right)}_{\max(1)} - A}_{\max(2)} \qquad (10)$$

Table 3 tries to summarise the differences between the two optimisation procedures.

Table 3. - Comparison of two procedures to optimally balance customer acquisition and retention costs

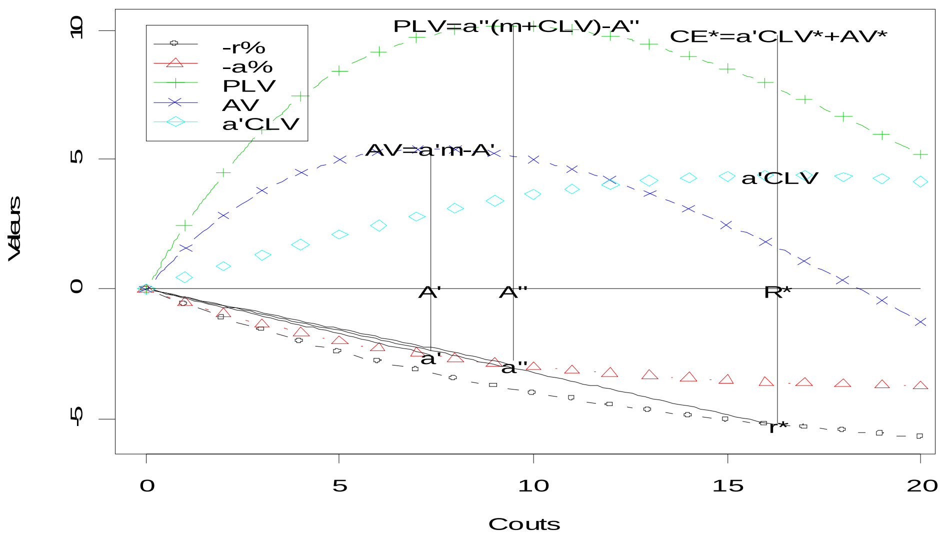
Compared elements	Blattberg et Deighton (1996)	Pfeifer(2005)
Perspective regarding the customer acquisition stage	Short term	Long term
Gains used to optimise acquisition value	Margin	CLV of a new customer
Calculation perspective	Prospect	Prospect
Customer value components (isolated)	Prospect acquisition value + just acquired CLV in a prospect perspective	Prospect Lifetime Value - Acquisition Cost
Formulation (same formula, different maximisation order)	$\underbrace{a(A)(m - A / a(A))}_{\max(1) \to a'} + \underbrace{a' CLV(R)}_{\max(2)}$	$\underbrace{a(A)\underbrace{(m + CLV(R))}_{\max(1)} - A}_{\max(2)}$
Stage 1	Find acquisition spending A* that maximises acquisition value of a prospect (on the short term) a(m-A/a) and consider the optimal rate a' obtained in this way.	Find retention spending R* that maximises CLV.
Stage 2	Find retention spending R* that maximises CLV.	Find acquisition spending A* that maximises PLV

In our opinion the two optimisation procedures have merits. The Blattberg and Deighton method looks for complete separation between the acquisition and retention stage and seems to adopt a risk adverse attitude to acquisition spending optimisation. This is a short term view which results in lower acquisition rate and spending as compared to the long term view suggested by Pfeifer (2005). The latter, by adopting a risk neutral attitude, regroups the two stages and integrates expected customer value in the calculation that aims to optimise acquisition spending. By replacing the margin with the lifetime value the latter method invests more in customer acquisition and potentially results in higher customer equity and lower optimal ratios of customer retention to acquisition**.**

Although we adopt Pfeifer's (2005) approach in our subsequent developments, we don't reject the original BD model and don't consider their optimisation procedure as being wrong. We prefer to regard it as an alternative trying to separate reality that can be measured (acquisition value) from the future that can be only be guessed or estimated (retention value) [iii].

By taking Blattberg and Deighton's original numeric example the comparison between the two procedures can be illustrated graphically as in figure 3.

Figure 3 Two procedures to find optimal balance between acquisition and retention spending compared



-r% = retention rate on a negative scale, **-a%** = acquisition rate on a negative scale, **PLV**=expected prospect Lifetime value, **CLV**=Lifetime value of a just acquired customer, **AV**=Acquisition value of a prospect, **CE***= optimal customer equity in a prospect perspective, **R***=optimal retention spending , **r***=optimal retention rate, **A'** = optimal acquisition spending (short term view as in Blattberg & Deighton, 1996), **A''**=optimal acquisition spending (long term view as in Pfeifer, 2005), **a'**=optimal acquisition rate (short term view), **a''**=optimal acquisition rate (long term view)

The parameters defining retention response behaviour are inferred from managerial experience. The response ceiling c_r =70% is directly assessed while the responsiveness parameter k_r is calculated knowing the retention rate r=40% obtained for a given retention effort per customer R=10$. It results in $k_r = \log(c_r/(c_r-0,4))/10= 0.0847$.

Figure 3 shows how the retention spending per targeted customer determines the response curve of the retention rate and indicates graphically the optimal spending R* that produces the highest CLV. Notice that this curve, that is represented on the negative scale, starts at the origin[iv], goes through the point ($10,0.4), and increases asymptotically to the ceiling rate (c_r). At optimal spending of 16.32$ the firm retains 52.4% of its customers at an average (or cost per retained customer) of $30.1 to achieve a maximum CLV* of $17.2 for a just acquired customer or $67.2 for a new customer if acquisition margin (m=50) is included.

Acquisition behaviour is similarly inferred in the original BD example, by directly assessing the acquisition response ceiling c_a =40% and by computing the responsiveness parameter k_a using the response rate *a=20%* for a given acquisition spending *A=5$* to be $k_a= log(c_a/(c_a-0,2))/5=0,13863$.

Figure *3* shows how acquisition spending per prospect determines acquisition rates and can indicate the optimal acquisition spending graphically. Here the original BD procedure differs from Pfeifer's (2005). By separating an acquisition value that ignores future gains from retention value, it finds smaller optimal acquisition spending. By spending an optimal $A'=(1/k_a)\ ln(k_a\ c_a\ m)\ =7.356\$$ a prospect acquisition rate of *a'=25.57%* is obtained at an average cost (or cost per acquired customer) of 49$ leading to a maximum prospect acquisition value a(m-A/a) of $5.43. By adding the just acquired customer lifetime value in a prospect's perspective (a' CLV*), optimal Customer Equity in a prospect's perspective is obtained (9.83$).

Pfeifer's approach considers not only present gains (margin) but also future gains (CLV) when maximising prospect acquisition value. This results in higher optimal acquisition spending $A'=(1/k_a)\ ln(k_a\ c_a\ CLV^*)$ = $9.49 to convert 29.2% of the prospects at an average cost of $32.42 to achieve an optimal prospect lifetime value PLV* of $10.18.

Besides the difference in the optimal amount to be spent on customer acquisition, in this particular numeric example the managerial recommendations differ fundamentally as to the optimal ratio between acquisition and retention spending a firm should adopt. The BD pessimistic approach which doesn't give future credit to acquisition recommends at optimality spending more on customer retention than on customer acquisition. Pfeifer's long term view gives more credit to acquisition and recommends in this specific example spending more on acquisition than on retention. This situation is illustrated graphically in figure 3. Optimal average acquisition and retention costs are ratios of these spending to the response rates they achieve. Graphically they are equivalent to cotangents of the acute angles starting in the origin (0,zero) and having as adjacent side of the triangle the optimal spending and as opposite side the response rate. The bigger this ratio (or the cotangent) is the bigger the angle. Optimal average retention spending is represented by the acute angle R^*0r^*. Optimal average acquisition spending is represented by angle $A'0a'$ for the original BD approach and by angle $A''0a''$ for Pfeifer's procedure. Compared to the angle indicating optimal average retention cost in figure 3, a bigger angle corresponding to the BD approach signifies smaller optimal average acquisition cost and recommends spending more on retention than on acquisition at optimality, while a smaller angle associated to Pfeifer's approach signifies bigger optimal average acquisition cost and recommends spending more on acquisition than on retention.

AN ANALYTIC SOLUTION TO THE BD PROBLEM

As mentioned before the retention response function used in the BD model (the modified exponential function) doesn't lead to a closed form expression for the optimal retention spending R*. Therefore the original authors suggest a graphic solution while other's like Pfeifer (2005) give numeric solutions (using Microsoft Excel's Solver). We introduce a response function that allows finding the optimal retention spending analytically with a closed form expression. The obvious advantage is quasi instantaneous calculation of the optimal marketing spending in the BD problem. This facilitates large scale sensitivity analysis and allows for a thorough investigation of optimal retention versus acquisition spending ratios under varying conditions.

This function applied to retention response is

$$r = c_r\left(1 - \frac{1}{1 + k_r R}\right) \tag{11}$$

Like the original retention response expression (the modified exponential function - formula 6) our function is consistent with an assumption of strictly diminishing returns to retention spending.

By introducing 11 in 3 and setting the derivative of the latter CLV expression by R equal to zero the retention spending that achieves highest customer lifetime value is given by:

$$R^{\mu} = \frac{-d-1+\sqrt{(1+d)c_r[(1+d-c_r)k_r m+1]}}{k_r(d+1-c_r)} \qquad (12)$$

In the appendix we provide a comparative list of indicators used in the Blattberg & Deighton model derived for the original modified exponential response function and for the proxy function being introduced[v].

This closed form expression we suggest might be easily applied in sensitivity analysis. It also helps deduce some limits or border conditions that should be observed in such situations. One such limit is the need to verify that optimal spending is positive. We show that this condition is satisfied for optimal retention spending when $c_r k_r > 1/m$ or $m > 1/(c_r k_r)$. Similarly in order to have a positive optimal acquisition spending the following condition needs to be satisfied $c_a k_a > 1/CLV^*$ or $CLV^* > 1/(c_a k_a)$. This is an interesting result as it links ceiling and responsiveness parameters to profitability and value measures. To illustrate lets take the original BD example. In order to have positive optimal retention spending the margin must be bigger than 1/(0.7* 0.0847)=16.87$ and in order to have positive optimal acquisition spending the maximum CLV must be bigger than 1/(0.4* 0.13863)=18$. An analyst can use these border conditions in several ways. He can, as already illustrated, find the minimum margin and optimum CLV that is needed in order to satisfy the border conditions knowing the prospects' and customers' responsiveness and response ceiling. He could also calculate border values for the responsiveness coefficients when knowing response ceiling, margin and optimal CLV. And finally he could compute response ceiling values border values when knowing responsiveness, margin and optimal CLV.

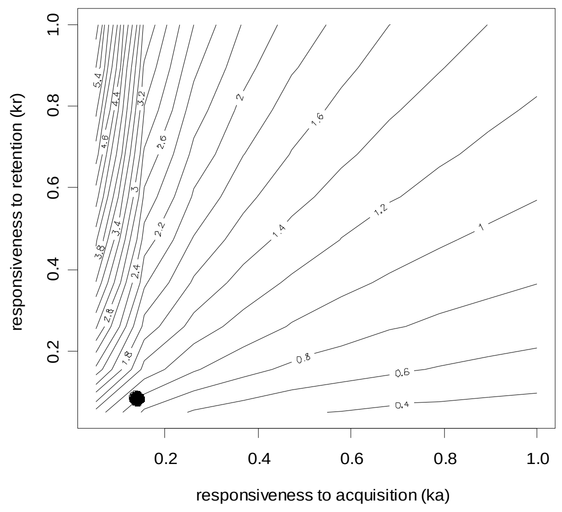
OPTIMAL ACQUISITION VERSUS RETENTION COST RATIO ANALYSIS

Analysing the optimal ratio between retention and acquisition costs provides further guidance to managers into whether it is optimal to spend more on retention when customers are cheaper to retain than to acquire. The ratio between retention and acquisition cost for a customer that marketing folklore generally fixes at 5 and for which empirical evidence (Sterne, 2003) suggests an interval varying from 3 to 20, should refer to costs per retained (R/r) and acquired (A/a) customer and not to marginal costs, as theoretically at optimality marginal retention and acquisition costs should be equal. We show that these average costs can be represented as cotangents in the graphical representation of the BD model, where optimal spending (A', A" or R*) is the adjacent side and the optimal response rate (a',a" or r*) is the opposite side of a triangle defining an acute angle whose size can then be visually used to evaluate the optimal ratio between retention and acquisition costs (see figure 3).

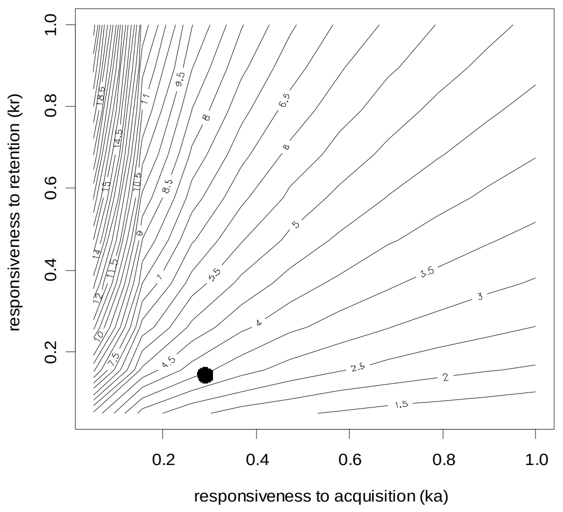
Pfeifer (2005) tries to prove that there is "no rule" that cheaper retention than acquisition costs imply spending more on retention by using a "carefully constructed" numerical example based upon BD's original example. We challenge this opinion and try to prove that there are rules. Cheaper to retain implies a combination of responsiveness (elasticity) and amplitude (ceiling – floor) in the customer response curve compared to the prospect's one. When the amplitudes of the two response curves are identical higher responsiveness in retention implies that it is optimal to spend more on retention and vice versa. Amplitudes can, when higher for retention than for acquisition (which is normally the case), strengthen this ratio in favour of retention if retention responsiveness is higher but can influence the opposite way when acquisition responsiveness is higher. This situation appeared in the original BD example and has been speculated by Pfeifer (2005). But his "carefully constructed example" is rather extreme and invalidates somehow the conclusions)[vi].

Instead of isolated "carefully constructed" numerical examples we use our analytic solution to the BD problem in order to produce a systematic decision support tool that uses iso-curves or contour lines that clearly show how varying pairs of acquisition and retention responsiveness parameters influence the optimal ratio among the two spending alternatives (see figure 4).

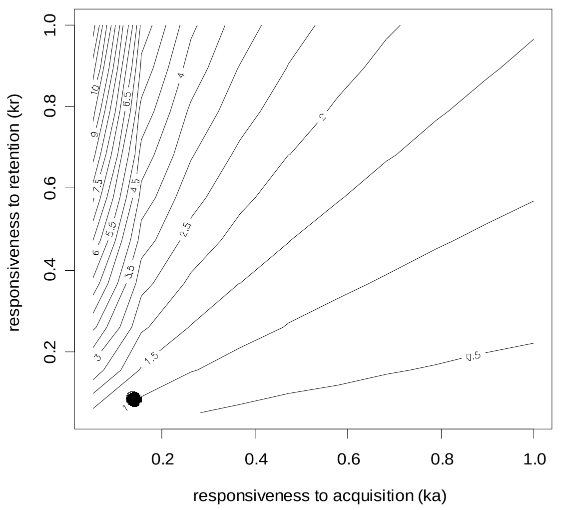
Figure 4 - Ratio of optimal average acquisition to retention spending for varying customer and prospect responsiveness



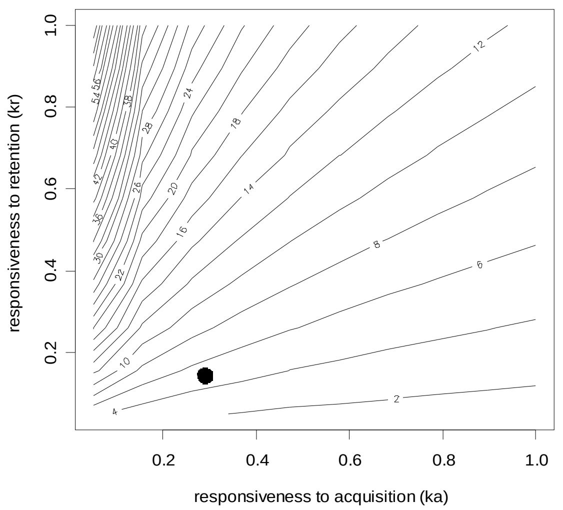
(a) normal response amplitudes (c_r=0.7 and c_a=0.4) – analytic solution



(b) extreme response amplitudes (c_r=0.95 and c_a=0.04) analytic solution



(c) normal response amplitudes (c_r=0.7 and c_a=0.4) – numeric solution



(d) extreme response amplitudes (c_r=0.95 and c_a=0.04) – numeric solution

Figure 4 shows the ratio of optimal average acquisition to retention spending for customer and prospect responsiveness varying systematically. Four situations are illustrated, two using our analytic solution for optimal retention spending (figure 4a and 4b) and two using the original numeric solution for comparison. For each solution the "normal" BD example (figure 4a and

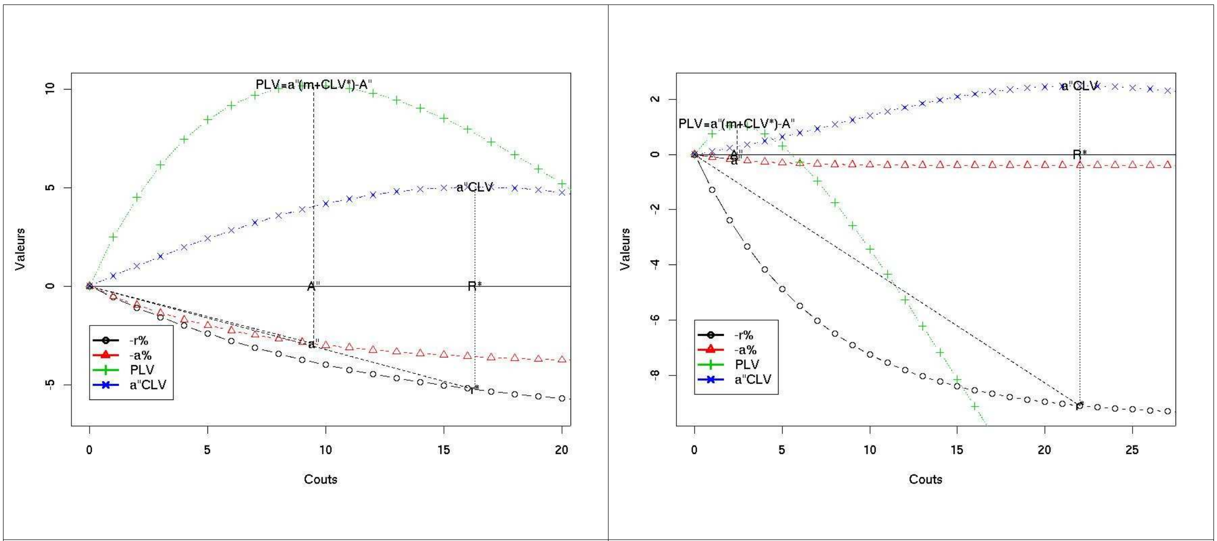
4c) and Pfeifer's extreme example (figure 4b and 4d) are fixing the response amplitudes. The filled circles indicate responsiveness parameter values for acquisition and retention corresponding to each example. The optimal ratio between acquisition and retention spending that is achieved by those responsiveness parameters can be read from the contour lines on each ratio map. For the original BD example one can see in figure 4c that the acquisition and retention responsiveness parameters were 0.13863 and 0.0847 respectively and that the resulting optimal ratio between acquisition and retention spending was slightly bigger than one. For the "extreme" Pfeifer (2005) example one can see in figure 4d that the acquisition and retention responsiveness parameters were 0.2888 and 0.1440 respectively and that at optimality the cost to acquire a new customer was five times (5x) the cost to retain an existing customer. This 5x ratio between optimal acquisition and retention costs is often mentioned in relationship marketing folklore.

For the normal situation our analytic solution (see figure 4a) gives a rather similar ratio map to the numeric solution (figure 4c). This proves that the new response function that has been introduced by us can be successfully used in "normal" situations as a "proxy" for the modified exponential retention response function and provide an easy and rapid way to compute the optimal ratio between average acquisition and retention spending. For "extreme" differences in response amplitude between acquisition and retention our analytic solution tends to understate the optimal ratio. Although the modified exponential function has been widely used in marketing, our function and the analytic solution it comes with doesn't need to be regarded as a "proxy". Its "satisficing" saturation and decreasing returns properties recommend it as a response function on its own that can be used as an alternative to represent customer retention response.

In order to produce optimal ratio contour maps with the numeric solution (figure 4a and 4b) we used powerful numeric optimisation algorithms available in S-plus and R systems (R Development Core Team, 2006).

Managers can use these contour maps corroborated with a decision support tool that would allow them to estimate either objectively or subjectively (by decision calculus) their customers' acquisition and retention response curves and find optimal acquisition and retention spendings like in figure 5.

Figure 5 – Visual decision support to optimal acquisition and retention spending



(a) normal response amplitudes (c_r=0.7 and c_a=0.4)	(b) extreme response amplitudes (c_r=0.95 and c_a=0.04)

-r% = retention rate on a negative scale, **-a%** = acquisition rate on a negative scale, **PLV**=expected prospect Lifetime value, **CLV**=Lifetime value of a just acquired customer, **AV**=Acquisition value of a prospect, **R***=optimal retention spending , **r***=optimal retention rate, **A''**=optimal acquisition spending , **a''**=optimal acquisition rate

After having estimated the acquisition (a) and retention (r) response curves as in figure 5a associated to the original BD example or as in figure 5b linked to Pfeifer's example, the manager obtains optimal acquisition and retention spendings where from optimal ratio between retention and acquisition spending can be derived. Graphically the optimal ratio is also given by the cotangent of the *optimal retention angle* (r^*0R^*) divided by the cotangent of the *optimal acquisition angle* $(a''0A'')$. One can easily see in figure 5a that retention angle is slightly bigger than the acquisition angle which gives an optimal retention to acquisition ratio near to one. This result can be identified on the contour map in figure 4a. On figure 5b the retention angle is much bigger than the acquisition angle. This ratio is near to 5 and can be identified on the contour map in figure 4b. In a segmented customer base, response to acquisition and retention efforts varies among segments and each segment can be positioned on a contour map. According to its responsiveness to acquisition and retention efforts the segment's optimal ratio can be read from the map. If response amplitude is similar among segments, then several segments can be positioned on the same contour map. Customer acquisition and retention policy can take profit from such decision support tools.

CONCLUSIONS AND FURTHER RESEARCH

The contributions of this research focus on the BD model and its extensions. This model consists essentially of two customer and prospect response models, of a simple customer dynamic behaviour model (the "retention model") and of an optimisation procedure for balancing customer acquisition and retention spending. As such it is a rather stylised model, well suited to explore the rationale that control marketing efforts to acquire new customers and retain existing ones. We adopt Pfeifer's (2005) modification of the BD optimisation procedure as it seeks a sound economic formulation to balancing customer acquisition and retention efforts. We don't go that far as to consider the BD optimisation procedure wrong and prefer to see it as a short term and pessimistic approach when finding optimal acquisition costs. We appreciate the BD model's approach to complexity reduction and separation of concerns.

Our analysis of the question whether it is optimal to spend more on retention when customers are cheaper to retain than to acquire, diverges from the one offered by Pfeifer. While Pfeifer considers that there is no rule and tries to prove it by two "carefully constructed" numerical examples, we build maps of optimal ratio between acquisition and retention spending as a decision support tool that enable us to show that there are rules. These rules are determined by a combination of responsiveness and amplitude in the customer response curve compared to the prospect's one. When the amplitudes of the two response curves are identical higher responsiveness in retention implies that it is optimal to spend more on retention and vice versa. Amplitudes can, when higher for retention than for acquisition (which is normally the case), strengthen this ratio in favour of retention if retention responsiveness is higher but can influence the opposite way when acquisition responsiveness is higher.

As for the original response functions in the BD model there is no closed form expression to compute optimal retention spending we used powerful numeric optimisation algorithms available in R/S-plus statistic systems in order compute the optimal customer acquisition versus retention spending ratio. Maps could be drawn representing ratio values corresponding to all combinations of retention versus acquisition responsiveness parameters under given response amplitudes. In this way the rules linking response amplitude and responsiveness to optimal ratio of retention to acquisition spending are visually revealed and can be used by managers and analysts.

An important contribution of this paper is the analytic solution to the BD problem. It uses a different response function that replaces the original modified exponential response functions. The closed form expression for optimal retention spending that results makes all calculations significantly quicker and accelerates the drawing of contour maps as a decision support tool.

Visual decision support artefacts like representing average retention and acquisition spending as cotangents of triangles that are added to the BD graphical solution can also to be considered as contributions of this paper.

By analogy to the analytic solution to the BD problem presented here, further research might try to develop proxy formulas enabling managers to approximate the optimal ratio between retention and acquisition spending by avoiding computing intensive numerical solutions. Some of the artefacts discussed or developed here in customer cohort context might be extended to the larger customer base and customer portfolio context.

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APPENDIX

Indicators used in the Blattberg & Deighton model derived for the original modified exponential response function and for the alternative function being introduced.

no	Indicator	modified exponential response	proxy response function
1	Retention response function	$r = cr(1 - exp(1 - krR))$	$r = c_r\left(1 - \frac{1}{1 + k_r R}\right)$
2	Inverse of the response function	$R = \frac{1}{k_r \ln\left(\frac{c_r}{c_r - r}\right)}$	$R = \frac{1}{k_r \left(\frac{c_r}{c_r - r}\right)}$
3	Average retention cost (2 divided by r)	$\overline{R} = \frac{1}{r\, k_r \ln\left(\frac{c_r}{c_r - r}\right)}$	$\overline{R} = \frac{1}{r\, k_r \left[\frac{c_r}{c_r - r}\right]}$
4	Marginal retention cost(derive 2 by r)	$R_r' = \frac{1}{k_r\,(c_r - r)}$	$R_r' = \frac{1}{k_r\,(c_r - r)} + \frac{r}{k_r\,(c_r - r)^2}$
5	Customer Lifetime Value	$CLV = (m - R / r(R))\frac{r(R)}{1 + d - r(R)}$	
6	Optimal retention cost (R that maximises 5)	R^* = numeric solver ..	$R^* + \frac{-d - 1 + \sqrt{(1+d)c_r[(1 + d - c_r)k_r m + 1]}}{k_r(d + 1 - c_r)}$
7	Acquisition response function	$a = c_a(1 - exp(1 - kaA))$	$a = c_a\left(1 - \frac{1}{1 + k_a A}\right)$
8	Inverse of the response function	$A = \frac{1}{k_a \ln\left(\frac{c_a}{c_a - a}\right)}$	$A = \frac{1}{k_a \left(\frac{c_a}{c_a - a}\right)}$
9	Average acquisition cost (8 divided by a)	$\overline{A} = \frac{1}{a\, k_a \ln\left(\frac{c_a}{c_a - a}\right)}$	$\overline{A} = \frac{1}{a\, k_a \left(\frac{c_a}{c_a - a}\right)}$
10	Marginal acquisition cost (derive 8 by a)	$A_a' = \frac{1}{k_a\,(c_a - a)}$	$A_a' = \frac{1}{k_a\,(c_a - a)} + \frac{a}{k_a\,(c_a - a)^2}$
11	Prospect Lifetime Value	$PLV = a(A)\,(m + CLV*) - A$	

12	Optimal acquisition cost (derive 11 to A)	$A^* = \frac{1}{k_a}\ln(k_a\ c_a\ CLV*)$	$A^* = \frac{1}{k_a}\ln(\sqrt{k_a\ c_a\ CLV*-1})$

i the distinction between the "lost for good" and "always a share" customer dynamic behaviour has been introduced by Jackson(1985) for industrial markets and extended by Dwyer to the consumer market. The first is associated to financial services, financial services, press subscriptions, etc. while the second is typical for catalogue buying.

ii Pfeifer et alii (2005) call this value "Expected Lifetime Value of a just acquired customer"

iii Some authors while not overtly rejecting CLV and financial analysis, consider that "valuation based on projections of cash flow is flawed both behaviourally (forecasting is inherently unreliable) and conceptually (because those forecasts include the results of future marketing)" (Ambler, 2003, p.57) and criticise regarding customers as assets: " Customers are not owned by their suppliers and they are not there to be milked." (ibid., p.58)

iv zero spending means zero retention, a limitation that can be easily overcome, as shown before

v Details on how formula 12 was derived can be obtained on demand from the authors.

vi We suppose there is a printing error in Pfeifer's (2005, p 186) example. These expected results can only be obtained if the acquisition response ceiling is ten times smaller than the one displayed. If this is true then this "carefully constructed" example is rather extreme and not very realistic, with a saturation level near to 100% in retention rate and near 0% in acquisition rate.